Cameron M. Weber
New School for Social Research
New York, USA
cameron_weber@hotmail.com
cameroneconomics.com

A Critique of Walrasian Equilibria in the Edgeworth Box[ϕ]

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[ϕ] The Edgeworth Boxes found here are based on material presented by Professor Duncan K. Foley and Teaching Assistant Luca Zamparelli during the Advanced Microeconomics course (Fall 2007) at the New School for Social Research in New York. This paper was to be part of a larger project evaluating what seems to be contradictions in Walras on the notions of equilibrium in his mathematics versus the narratives, the latter which use a more entrepreneurial market process of trial and error in price discovery to theorize the economy. Walras' narratives are more aligned with the Austrian School in the marginal revolution of the 1870s than the way Walras has been received to date in economics with the "one price" equilibrium axiom for clearing markets. This larger project is put on hold due to the opportunity cost of time.

I. The Edgeworth Box, Trading Paths and the Concept of Equilibrium

In this short paper we illustrate and critique the notions of Pareto efficiency and Walrasian equilibrium using the Edgeworth Box to show the concept of trading paths leading to equilibrium. We present two different models of Pareto efficient trading outcomes;

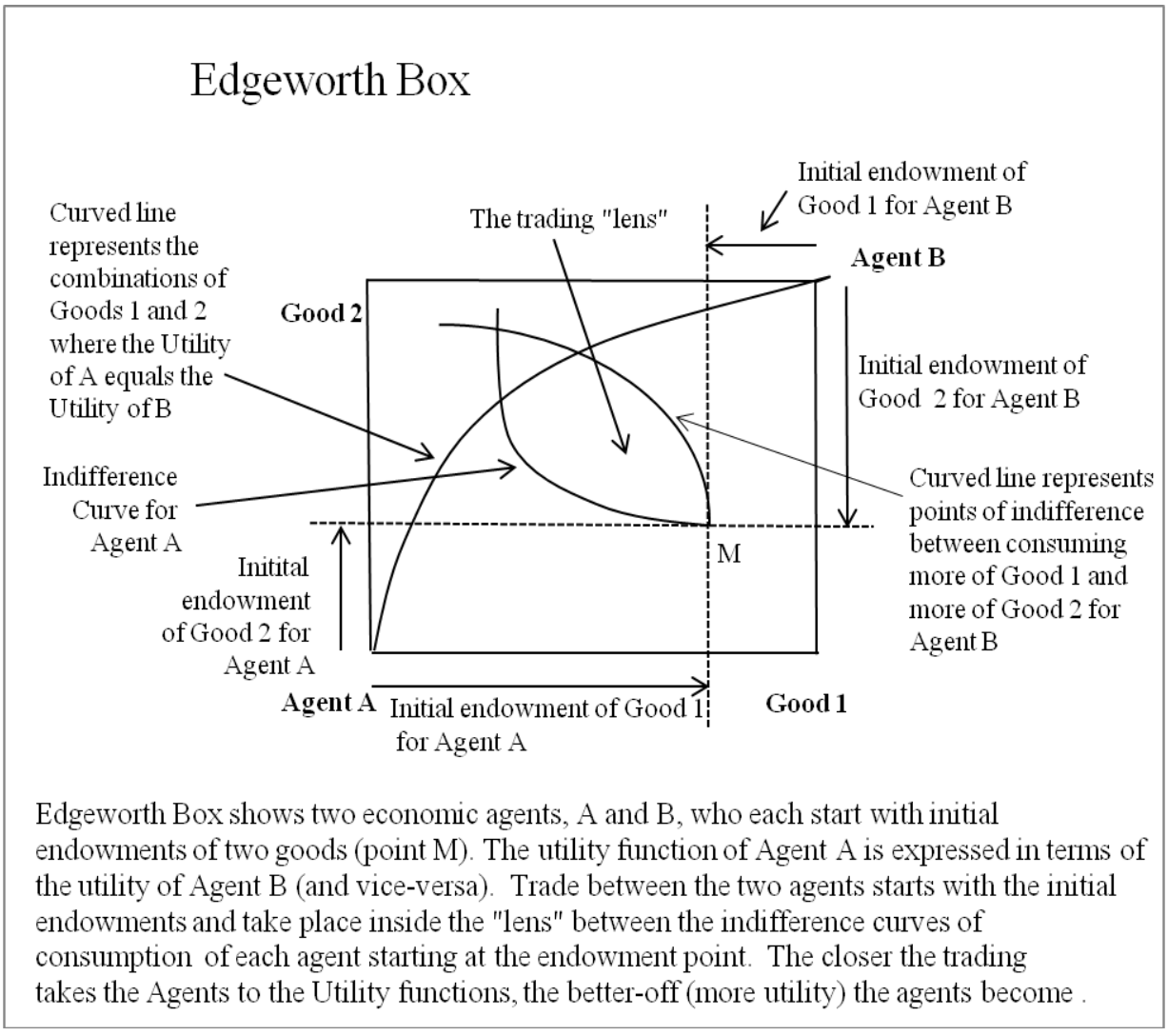
1) Trading equilibrium where trades are decentralized and random (incomplete information), and,
2) Walrasian equilibrium where there is just one market clearing price for all trades (full information).

Pareto did not have a concept for equilibrium other than the notion of efficiency where different individuals traded to maximize their utilities.[1] He described a "set" of trades where different economic agent's utilities were equal because their Indifference Curves (their preferences for the consumption of different bundles of goods) were tangent to each other. A Pareto Efficient outcome, in other words, is where agents are trading at a point where neither could be made better without making the other worse off.

In Illustration 1. we find the basic Edgeworth Box, containing the initial endowments of Agents A and B and the trading lens inside which trades are possible (no agent would trade outside of the lens because it would mean they would move to a lower Indifference Curve, e.g., a lower consumption bundle than their initial endowment of goods). We also find the contract curve where the utility functions of each agent are equal inside the trading lens; the points along the contract curve are the possible points of trading equilibria.

[1] "Pareto held that the utilities of different persons were incommensurable" (Little 1956, p. 85)

Illustration 1.



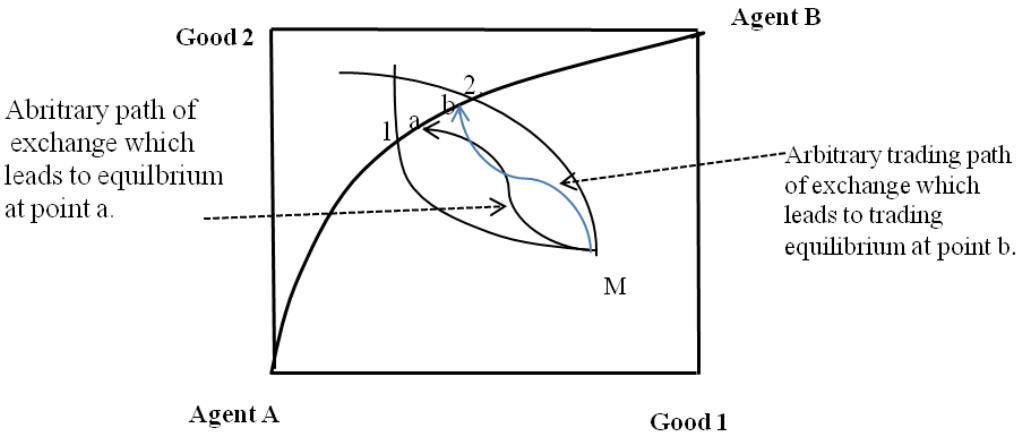
Edgeworth Box shows two economic agents, A and B, who each start with initial endowments of two goods (point M). The utility function of Agent A is expressed in terms of the utility of Agent B (and vice-versa). Trade between the two agents starts with the initial endowments and take place inside the "lens" between the indifference curves of consumption of each agent starting at the endowment point. The closer the trading takes the Agents to the Utility functions, the better-off (more utility) the agents become .

As a heuristic device Foley (2007) introduced the concept of "trading equilibria" which shows how different trading paths can lead from the initial endowments of agents to different Pareto efficient outcomes.[2] This is shown below in Illustration 2. You can see that under the trading path which leads to outcome (a.) that Agent B has a better distribution outcome because Agent B ends up with more of both Good 1 and Good 2 than does Agent A. Yet no more trades are possible because for Agent A to gain more of Good 1 or Good 2, Agent B would have to give up some Good 1 or Good 2. The opposite is true of the trading path leading to outcome (b.). Both of these trading equilibria are Pareto efficient because no one agent can gain without a loss to the other agent.

[2] Foley's innovation in the Edgeworth Box addresses the original critique of Walras by Bertrand 1883 (in Mirowski 1989, pp. 241-243) which states that Walras' mathematics misses the notion of 'false trading' (or in Austrian School language, the price discovery process through decentralized markets) and that the now mainstream ("neo-classical") equilibrium axiom based on Walras' "one-price" as an overly simplified analogy for the economy. Mirowski; "speculation.., would obviate the determinacy of Walras' general equilibrium," p. 242.

Illustration 2.

Edgeworth Box showing arbitrary trading paths toward trading equilibia

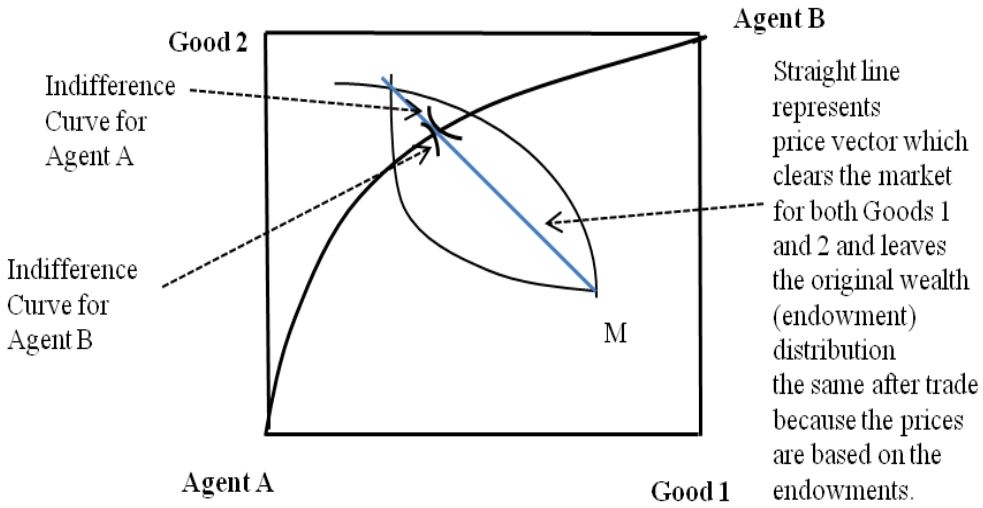


Trading equilbrium is between points 1. and 2. inside the trading lens along the curved line where the Utility functions of Agent A and B are equal. It is along this trading equilbrium where the propensity to consume each good in relation to the other good is the same for each agent (the Marginal Rates of Substitution are equal) so agents will tend to trade to gain economic surpluses until they are on this trading equilbrium set, also known as the Pareto Set, the contract curve or the efficiency frontier. There is no mathematical theorem under Welfare economics nor other economic theory, other than the "market discovery process", which would dictate the trading paths towards a trading equilibrium.

Illustration 3. shows the mainstream neo-classical approach to Walrasian equilibrium, where a price vector is pre-determined before any trades are made and then once prices are determined, the Agents immediately move (trade) to reach the point of tangency between the two agents' Indifference Curves on the contract curve (also known as the Pareto Set or Efficiency Frontier).

Illustration 3.

Edgeworth Box showing Walrasian Equilibrium with prices set by benevolent Auctioneer



The Walrasian Equilbrium is found inside the trading lens at the point of tangency between the Indifference Curves of Agent A and Agent B. The price vector, p*, goes from the endowment point, m, straight to the point tangency of the Indifference curves on the contract curve and again does not define a trading path. Once the p* is determined the agents immediately trade, moving to the Pareto efficient equilibrium. It could be proposed that there are many p* for each incremental trade that the agents make in the economy, and that the Walrasian equilbrium represented here is just an anology for many continuous trades in an economy, which would be a more realistic model than assuming an immediate jump from the initial endowments to the final point of equilibrium. However, this is not how Walras' writings have been interpreted in mainstream neo-classical economic theory.

The most-often used "axiom" to arrive at the Walrasian equilibrium (again defined where there is a set of prices which fully matches all buyers and sellers of an economy's goods leaving no excess demand and no further trades making anyone better off without making someone else worse off) is what is known as the (benevolent) Auctioneer[3].

> To capture the logic of the Walrasian assumptions, imagine a third party – called the Auctioneer – whose job it is to suggest price ratios at which we might find trade and the ensure that no trading takes place until prices are found such that market clear. The Auctioneer simply announces various prices, and for each price we indicate how much of one good we are willing to exchange for the other. This hypothetical process continues until a market-clearing price is hit upon (that is a price is found such that my desired purchase of your Y are exactly offset by your desired sales of Y, and similarly for the other good. Under reasonable assumptions, there is at least one price ratio that will accomplish this, and when it is found, market-clearing trades take place and the resulting allocation – called the competitive equilibrium – will be Pareto efficient (Bowles, p. 212).

[3] There were three people credited with creating the" marginal revolution" in economics which broke away from the classical economist's concept of the value of goods sold in the economy being based on costs of production – these costs (mostly labor) were seen as a natural price around which the market price would fluctuate. The marginalists (Jevons, Menger and Walras, all writing in 1871-1873) stated that the value of a good was based on the subjective preferences of consumers. Menger, the founder of the Austrian School, did not have a physics-based theory of equilibrium like Walras and Jevons but rather stated the economy was decentralized and the price discovery process, not a "one-price" equilibrium clearing, was the coordinating mechanism of the market. Whereas Walras used the "auctioneer" to justify how the market came to equilibrium, Jevons "invented a black box, a 'trading body', which magically performed all the dynamic functions of coordination in an unspecified manner," Mirowski 1989, p. 251. Also, see Mirowski 1989, pp. 193-276, on the necessary condition of "one-price" in equilibrium economics.

Appendix

Chronology of Major Works Informing Modern Mainstream Neo-Classical Welfare Economics

Bentham, J. 1789. *The Principles of Morals and Legislation*

Walras, L. [1874] 1926 (and various editions). *Elements d'Economie Politique Pure.*

Edgeworth, F.Y. 1881. *Mathematical Psychics.*

Pareto,V. 1909. *Manual of Political Economy.*

Pigou, A.C. 1920. *The Economics of Welfare.*

Arrow, K.J. and Debreu G. 1954. "Existence of an Equilibrium for a Competitive Economy." *Econometrica* 22:256-291.

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